Question 3 **(7 marks)**

Consider the function $f(x) = \dfrac{(x-1)^2}{e^x}$.

(a) Show that the first derivative is $f'(x) = \dfrac{-x^2 + 4x - 3}{e^x}$. (2 marks)

(b) Use your result from part (a) to explain why there are stationary points at $x = 1$ and $x = 3$. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3 (continued)

It can be shown that the second derivative is $f''(x) = \dfrac{x^2 - 6x + 7}{e^x}$.

(c) Use the second derivative to describe the type of stationary points at $x = 1$ and $x = 3$.
(3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 4 **(7 marks)**

Consider the function defined by $f(x)=\dfrac{x}{2}-\sqrt{x},\ x\geq 0$.

(a) Determine the coordinates of the stationary point of $f(x)$. (3 marks)

(b) Use the second derivative test to determine the nature of the stationary point found in (a).(3 marks)

(c) State the global minimum of $f(x)$. (1 mark)

Question 3 **(7 marks)**

Consider the function $f(x)=\dfrac{(x-1)^2}{e^x}$.

(a) Show that the first derivative is $f'(x)=\dfrac{-x^2+4x-3}{e^x}$. (2 marks)

Solution
$f'(x)=\dfrac{e^x 2(x-1)-e^x(x-1)^2}{e^{2x}}$ $=\dfrac{e^x(x-1)(2-x+1)}{e^{2x}}$ $=\dfrac{-(x-1)(x-3)}{e^x}$ $=\dfrac{-x^2+4x-3}{e^x}$
Specific behaviours
✓ uses quotient rule ✓ simplifies expression

(b) Use your result from part (a) to explain why there are stationary points at $x=1$ an $x=3$. (2 marks)

Solution
$f'(x)=\dfrac{-(x-1)(x-3)}{e^x}$ $f'(1)=0=f'(3)$
Specific behaviours
✓ identifies stationary points as $f'(x)=0$ ✓ shows that this is true for $x=1,3$

It can be shown that the second derivative is $f''(x) = \dfrac{x^2 - 6x + 7}{e^x}$.

(c) Use the second derivative to describe the type of stationary points at $x = 1$ and $x = 3$. (3 marks)

Solution
$f''(x) = \dfrac{x^2 - 6x + 7}{e^x}$ $f''(1) = \dfrac{2}{e}$ $f''(3) = \dfrac{-2}{e^3}$ when $x = 1$ $f'' > 0$ hence local minimum when $x = 3$ $f'' < 0$ hence local maximum
Specific behaviours
✓ evaluates second derivatives for $x = 1$ and $x = 3$ ✓ uses sign to determine nature ✓ states nature for each stationary point

Question 4 **(7 marks)**

Consider the function defined by $f(x)=\frac{x}{2}-\sqrt{x},\ x\geq 0$.

(a) Determine the coordinates of the stationary point of $f(x)$. (3 marks)

Solution
$f'(x)=\frac{1}{2}-\frac{1}{2\sqrt{x}}$ $\frac{1}{2}-\frac{1}{2\sqrt{x}}=0 \Rightarrow x=1$ $f(1)=\frac{1}{2}-\sqrt{1}=-\frac{1}{2}$ $\Rightarrow$ stationary point at $\left(1,\ -\frac{1}{2}\right)$
Specific behaviours
✓ differentiates function ✓ solves $f'(x)=0$ ✓ states coordinates of point

(b) Use the second derivative test to determine the nature of the stationary point found in (a). (3 marks)

Solution
$f''(x)=\frac{1}{4\sqrt{x^3}}$ $f''(1)=\frac{1}{4}$ $f''(1)>0$ $\Rightarrow$ local minimum
Specific behaviours
✓ determines second derivative ✓ shows $f''(1)>0$ ✓ states conclusion that point is local minimum

(c) State the global minimum of $f(x)$. (1 mark)

Solution
$-\frac{1}{2}$
Specific behaviours
✓ states correct value of global minimum